VISCOSITY-GRAVITATIONAL FLOW IN TUBES WITH A SMALL EFFECT OF THERMOGRAVITATION

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There is a considerable body of work devoted to investigation of viscosity-gravitational flows in tubes. However, all the calculating recommendations in these communications relate to the case of stabilized flow. At the same time, in practically all real objects, the development of the process takes place in the initial section, since, for the onset of stabilization, a great length of the tubes is required.

In the present work an analytical investigation was made of the limits and the character of the start of the effect of thermogravitational forces on the fields of the velocity and the temperature, the friction resistance, and the heat transfer in any arbitrary cross section along the length of round tubes, arbitrarily arranged in space, with a constant density of the heat flux at the wall (q_w = const).

We use the equation of motion, written for the curl

$$d\boldsymbol{\omega}/d\tau = (\boldsymbol{\omega}\nabla)\mathbf{u} + \boldsymbol{\nu}\Delta\boldsymbol{\omega} + \operatorname{curl}\rho\mathbf{g}, \tag{1}$$

where $\boldsymbol{\omega} = \operatorname{curl}\mathbf{u}$; $\mathbf{u} = v\mathbf{e}_r + w\mathbf{e}_\varphi + u\mathbf{e}_x$; $\mathbf{g}$ is the acceleration due to gravity; and $\boldsymbol{\nu}$ is the kinematic viscosity coefficient.

The problem is solved with the following premises: 1) the process is steady-state; 2) the physical properties of the liquid are constant with the exception of a change in the density, taken into consideration in the term of the mass forces; the dependence of the density on the temperature is represented in the form $\rho = \rho_{w,0}[1 - \beta(t - t_{w,0})]$; the coefficient of volumetric expansion β is assumed constant; 3) in the cross section corresponding to the start of heating, the parabolic distribution of the velocity $u_l = 2\bar{u}(1 - R^2)$ is given as fully established; 4) we consider viscosity-gravitational flow with a weak effect of thermogravitational forces, i.e., a small deviation from the values characteristic for viscous flow is sought; 5) the change in the parameters along the flow is considerably less than over the radius; the problem is solved with a boundary condition of the second kind $q_w = -\lambda\partial t/\partial r|_{r=d/2}$ = const.

In solution of the problem, in addition to Eq. (1), use is made of the equation of continuity, written in dimensionless form

$$\frac{\partial}{\partial R}(RV) + \frac{\partial W}{\partial\varphi} = 0 \tag{2}$$

and the linearized equation for a small deviation of the temperature

$$\frac{U'}{4}\frac{\partial\Theta_l}{\partial X} - \frac{\mathrm{Pe}}{2}V\frac{\partial T_l}{\partial R} = \frac{1}{R}\frac{\partial}{\partial R}R\frac{\partial\vartheta}{\partial R} + \frac{1}{R^2}\frac{\partial^2\vartheta}{\partial\varphi^2}, \tag{3}$$

where $R = 2r/d$; d is the diameter of the tube; $V = v/\bar{u}$ is the radial component of the velocity; $W = w/\bar{u}$ is the tangential component; $\bar{u}$ is the mean velocity; and $\vartheta = (t - t_l)\lambda/q_w d = [(t_{w,l} - t_l)\lambda/q_w d] - [(t_{w,l} - t)\lambda/q_w d] = T_l - T$.

For solution of the problem, we need to know the field of the temperature with viscosity flow T_l along the whole length of the tube. The results of numerical [1] and analytical [2]

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solutions with constant physical properties and q_w = const are correlated in the interval $5\cdot10^{-4} < X < \infty$ in the form of the following interpolational dependences:

for $R \geq R_m$

$$T_l = T_m\left(1 - \frac{4}{3} R_*^{2+\alpha} + \frac{1}{3} R_*^{4+2\alpha}\right), \tag{4}$$

where

$$T_m = (3/8)[1 - \exp(-78X)];\ R_* = (R - R_m)/(1 - R_m);$$
$$R_m = \exp(-14\sqrt{X});\ \alpha = 0{,}03X^{-2/3};$$

for $R \leq R_m$

$$T_l = T_m, \tag{5}$$

where $X = x/Ped$; $Pe = Pr\cdot Re$ is the Péclet number; $Re = \bar{u}d/\nu$ is the Reynolds number; Pr is the Prandtl number; and $U' = (U - U_l) = u'/\bar{u}$ is the deviation of the axial component of the velocity.

The dimensionless temperature Θ_l is described by the expression

$$\Theta_l = \frac{t_b - t_+}{q_w d}\lambda + \frac{t_{w,l} - t_b}{q_w d}\lambda - \frac{t_{w,l} - t_l}{q_w d}\lambda = 4X + \frac{1}{\mathrm{Nu}_l} - T_l, \tag{6}$$

where t_b is the mean-mass temperature of the liquid in a given cross section; t_+ is the temperature of the liquid in the inlet cross section; t_w is the temperature of the wall; t_l is the temperature of the liquid with laminar flow; and $Nu = q_w d/\lambda(t_w - t_b)$ is the Nusselt number.

In accordance with the assumptions given in [3], the Nusselt number Nu_l is satisfactorily described by the expressions

$$1/\mathrm{Nu}_l = X^{1/3}/1{,}31(1 + 2X) \text{ for } X < 0.037,$$
$$\mathrm{Nu}_l = 4.36 \text{ for } X > 0.07, \tag{7}$$

where Nu_l is the Nusselt number with laminar flow.

It is not possible to obtain a solution in general form using dependences (4)-(7). Therefore, solutions are sought for six values of the reduced length $X = x/Ped$ ($X = 6.5\cdot10^{-4}$; 10^{-3}; $1.84\cdot10^{-3}$; $5.2\cdot10^{-3}$; $1.47\cdot10^{-2}$; $X > 0.07$), corresponding to the values α = 4, 3, 2, 1, 0.5, and 0.

In accordance with the assumptions adopted, Eq. (1) in projection on the axes of cylindrical coordinates in dimensionless form is written in the form

$$\frac{d^2}{dR^2}\Omega_\varphi + \frac{1}{R}\frac{d}{dR}\Omega_\varphi - \frac{\Omega_\varphi}{R^2} = \frac{\mathrm{Gr}}{4\,\mathrm{Re}}\frac{\partial T_l}{\partial R}\cos\psi; \tag{8}$$

$$\frac{1}{R}\frac{\partial}{\partial R} R\frac{\partial\Omega_x}{\partial R} + \frac{1}{R^2}\frac{\partial^2\Omega_x}{\partial\varphi^2} = \frac{\mathrm{Gr}}{4\,\mathrm{Re}}\frac{\partial(RT_l)}{R\partial R}\sin\varphi\cdot\sin\psi; \tag{9}$$

$$\frac{1}{R}\frac{\partial}{\partial R} R\frac{\partial\Omega_r}{\partial R} + \frac{\partial^2\Omega_r}{R^2\partial^2\varphi} = \frac{\mathrm{Re}}{2}\frac{dU_l}{RdR}\frac{\partial V}{\partial\varphi}, \text{ where} \tag{10}$$

$$\Omega_\varphi = \frac{\omega_\varphi d}{2\bar{u}} = \Omega_{\varphi,l} + \Omega_\varphi' = -\partial U/\partial R,\ U = U_l + U'; \tag{11}$$

$$\Omega_x = \Omega_x' = \frac{1}{R}\left[\frac{\partial}{\partial R}(RW) - \frac{\partial V}{\partial\varphi}\right],\ W = W',\ V = V'; \tag{12}$$

$$\Omega_r = \Omega_r' = \frac{1}{R}\,\partial U'/\partial\varphi, \tag{13}$$

where φ is the angle around the periphery of the tube, reckoned from the upper generatrix; ψ is the angle between the axis and the vertical; and $Gr = g\beta q_W d^4/\lambda\nu^2$ is the Grashof number.

The system of equations (2), (3), (8)-(10) is solved with the following boundary conditions:

for R = 0 all the values are finite; for R = 1, U = 0, V = 0, W = 0, $\partial\vartheta/\partial R = 0$; for $R = R_m$, all the functions and their derivatives are equal, respectively, to ("linking" of solutions for the core and prewall regions)

$$\text{for } \varphi = \pi/2 \text{ , } \int_0^1 W dR = 0;$$

$$\text{for } \psi = 0 \text{ and } R = 0 \text{ , } \partial U/\partial R = 0,\ \partial\vartheta/\partial R = 0;$$

$$\int_0^1 RU' dR = 0 \text{ or } 2\int_0^1 RU dR = 1.$$

Solving Eq. (8) in accordance with the recommendations of [4], and using the above boundary conditions, from the definition (11) we obtain an expression for the fraction of the axial component of the velocity, due to the action of axial forces:

$$U_a = U_l + U_a' = 2(1-R^2) + \frac{Gr\,T_m}{12\,Re}\Big\{0.75 R_m^2 (2 - R_m^2) \times \times (1-R^2) + 2(1-R^2)\int_{R_m}^1 R^2 \Gamma dR - \int_{R_m}^1 \Gamma dR + + \left[\begin{matrix} 0.75R^2 - 0.75R_m^2(1 - 2\ln R_m) & \text{for} & R \leqslant R_m \\ \left(1.5 R_m^2 \ln R + \int_{R_m}^R \Gamma dR\right) & \text{for} & R \geqslant R_m \end{matrix}\right]\Big\}\cos\psi, \tag{14}$$

where $\Gamma = f(R, X)$.

The results of a numerical calculation using Eq. (14) are approximated for all values of X and R by the interpolational equation

$$U_a = 2(1-R^2) \mp 2.25\cdot 10^{-3}\frac{Gr}{Re}\,[1 - \exp(-72X)^{3/2} \times \times \{1 - [1.1(2-R)^{3/2}\sin^2 0.6\pi R^2 + 0.1\sin\pi R]\ [\![1 + [1 - 1.5\exp(-150X)](1-R^2)]\!]\}\cos\psi. \tag{15}$$

We shall seek the component of the curl Ω_x from Eq. (9) in the form of the product

$$\Omega_x = A_x(R,\ X)\sin\psi\cdot\sin\varphi.$$

After substitution of this expression into Eq. (9), we obtain an equation with respect to A_x, analogous to (8).

After finding the distribution of Ω_x for the above six values of X, the distribution of the tangential W and radial V components of the velocity is sought from the relationship (12) and the equation of continuity (2). Under these circumstances, for W a substitution analogous to that for Ω_x is used, and an equation of the form of (8), into whose right-hand side the function Ω_x is again solved.

The calculated distributions of V are approximated by the interpolational equation

$$V = -(Gr/800Re)[1 - \exp(-12.5\sqrt{X})](1 - R^2)^4\cos\varphi\cdot\sin\psi.$$

Figure 1 shows the distribution of the tangential component of the velocity in the horizontal diametral plane and the radial component of the velocity in the vertical diametral plane with $\psi = \pi/2$ and values of $X = 5.2\cdot 10^{-3}$ (curve I), $X > 0.07$ (curve II) for the case of a heated wall. The distribution of the components of the velocity shows that the secondary free convective currents form a system of two longitudinal eddies with ascending near the

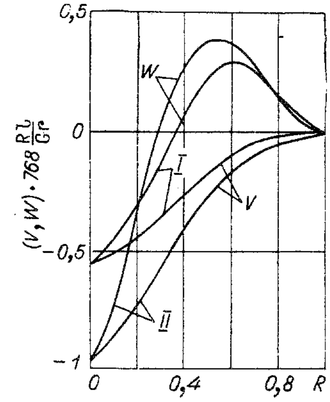

Fig. 1

wall and descending flow near the axis. With exactly the same value of Gr/Re, the intensity of the secondary currents decreases with a decrease in X. In the case of cooling of the wall, the direction of the secondary currents is the contrary.

Using the substitution $\Omega_r = \gamma \sin \varphi \times \sin \psi$, Eq. (10) can be transformed, with respect to γ, to a form analogous to (8). From (13) we find the deviation of the axial component of the velocity U_v' due to secondary flows,

$$U_v' = \frac{\mathrm{Gr}}{800}\left[1 - \exp\left(-12.5\sqrt{X}\right)\right] R^2 (-0.37 + 0.667R - 0.532R^2 + \\ + 0.342R^5 - 0.128R^7 + 0{,}02R^9)\cos\varphi \cdot \sin\psi. \quad (16)$$

Since the problem is being discussed within the limits of a linear approximation, the total deformation of the axial component of the velocity U' can be represented in the form of the sum of two terms described by relationships (15), (16),

$$U = U_l + U' = U_l + U_a' + U_v'. \quad (17)$$

We note that the parameter characterizing the effect of free convection on forced flow is different for different components of the velocity and for a different orientation of the tube with respect to the field of the force of gravity. Thus, the parameter Gr/Re enters into the description of the distribution of the tangential and radial components of the velocity, while the parameters Gr/Re and Gr enter into the description of the axial component. For the two extreme positions of the tube, the deformation of the axial component of the velocity is described by different parameters: with a vertical position ($\psi = 0$), by Gr/Re; with a horizontal position ($\psi = \pi/2$), by Gr. The change in the parameters describing viscosity-gravitational flow and the heat transfer with different positions of the tube was pointed out in [5], which discussed an analogous problem for stabilized conditions. Figure 2 shows the deformation of the axial component of the velocity in the vertical diametral plane, calculated using dependences (15)-(17) for four positions of the tube, starting from the vertical ($\psi = 0$) and ending with the horizontal ($\psi = \pi/2$). The calculation was made for the values $\mathrm{Gr} = 2\cdot10^4$, (Gr/Re) = 200, with values of the reduced length $X = 5.2\cdot10^{-3}$ (curves 1) and $X > 0.07$ (curves 2). The distribution of the velocity with viscosity-gravitational flow is compared with the parabolic profile with laminar flow (curves 3). Figure 2a, b shows the cases of flow upstream and downstream in a heated tube.

With a vertical position of the tube, the deformation of the profile of the velocity is described by the parameter Gr/Re. With an increase in the angle ψ between the vertical and the axis of the tube, secondary flows start to develop and the deformation of the profile of the velocity U is described both by the parameter Gr/Re and by Gr. In the case of a horizontal position, the deformation of U is described by the secondary flows, and is characterized only by the value of the Gr number. The secondary flows lead to an increase in the velocity in the lower part of the tube ($\varphi = \pi$).

Using the relationships

$$\Omega_\varphi|_{R=1} = -\left.\frac{dU}{dR}\right|_{R=1} = \frac{\tau_w}{\mu}\frac{d}{2\bar{u}}, \quad \langle\tau_w\rangle = 2\int_0^\pi \tau_w d\varphi,$$

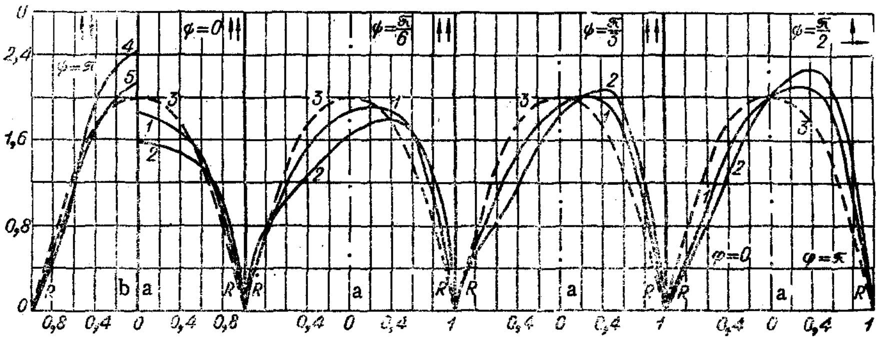

Fig. 2

we obtain an expression determining the change in the mean friction stress at the wall (τ_w) over the periphery and, consequently, also the coefficient of the friction resistance ξ:

$$\frac{\langle\tau_w\rangle}{\tau_{w,l}} = \frac{\xi}{\xi_l} = 1 \pm \frac{\mathrm{Gr}\, T_m}{48\,\mathrm{Re}}\left[-\Gamma|_{R=1} + 1.5R_m^2(1-R_m^2) + 4\int\limits_{R_m}^{1} R^2\Gamma dR\right]\cos\psi = 1 \pm \frac{\mathrm{Gr}}{\mathrm{Re}}\,\lambda(X)\cos\psi.$$

Values of $\lambda(X)$ were calculated for the six previously mentioned values of X and are approximated by the interpolation equation

$$\lambda(X) = 2\cdot 10^{-3}\,[1-\exp(-126X)]^{3/2}.$$

We obtain the deviation of the temperature ϑ from Eq. (3) in the form

$$\vartheta = \omega_1\cos\varphi\cdot\sin\psi + \omega_2\cos\psi. \tag{18}$$

After substitution of (18) into Eq. (3), by solution of two independent differential equations with respect to ω_1 and ω_2 and approximation of the corresponding solutions in the interval $5\cdot 10^{-4} < X < \infty$, the following expressions are obtained for ω_1 and ω_2:

$$\omega_1 = 9.1\cdot 10^{-6}\,\mathrm{Ra}\,[1-\exp(-100X)]^3\begin{Bmatrix} k_1 R \text{ for } R \leqslant \dfrac{1+k_2}{k_1+k_2} \\ 1+k_2(1-R) \text{ for } R \geqslant \dfrac{1+k_2}{k_1+k_2}\end{Bmatrix},$$

where

$$k_1 = 1 + 2.7[1-\exp(-48X)];$$

$$k_2 = 0.38[1-\exp(-10\sqrt{X})];$$

$$\omega_2 = 5.5\cdot 10^{-4}\frac{\mathrm{Gr}}{\mathrm{Re}}\begin{Bmatrix}(a_0 - a_2R^2 + a_4R^4 - 0.4R^6 + 0.025R^8) \text{ for } R \geqslant R_m \\ (b_0 - b_2R^2 + b_4R^4) \text{ for } R \leqslant R_m\end{Bmatrix},$$

where

$$a_0 = 0.086[1-\exp(-40X)];\ a_2 = [1-0.65\exp(-40X)];$$

$$a_4 = 1.05[1-0.31\exp(-40X)];\ b_0 = 0.086[1-\exp(-30X)];$$

$$b_2 = [1-\exp(-65X)];\ b_4 = 0.25[1-\exp(-65X)].$$

Relationship (18) shows that the deformation of the temperature field is determined by the parameters Ra and Gr/Re. Here, in the case of a horizontal position of the tube, the determining parameter, as for the velocity profile, is Gr/Re. In the case of a horizontal position, the deformation of the temperature field is determined by the number Ra = Pr Gr, and the deformation of the distribution of the axial component of the velocity, by the Gr number. Thus, depending on the value of the Pr number, with a horizontal (as well as with an inclined, although to a lesser degree) position of the tube, the most considerable factor will be the change in the profile of the velocity or the profile of the temperature.

In [6], on the basis of experimental data obtained with viscosity-gravitational flow of liquid in a horizontal tube, with a Prandtl number $Pr \approx 80$, it is shown that, with exactly the same value of the Gr number, the deformation of the temperature profile is considerably stronger than the deformation of the velocity profile. Experimental data [7] on the profiles of the velocity and the temperature with viscosity-gravitational flow of air ($Pr = 0.7$) show that the degree of deformation of the profiles of the velocity and the temperature is approximately identical. These results are in qualitative agreement with the results obtained in the present work. Unfortunately, it is not possible to compare the results of calculations with the experimental data, since the experimental data were obtained with a strong effect of thermogravitation. From this there follows the conclusion that from the temperature profile it is impossible to judge the degree of the effect of thermogravitation in a flow of liquid metal with $Pr \ll 1$.

The value of ϑ at the wall determines the increase in the dimensionless temperature of the wall (consequently, also the Nu number), i.e.,

$$\vartheta|_{R=1} = \vartheta_w = (t_w - t_{w,l})\lambda/q_w d = 1/\mathrm{Nu} - 1/\mathrm{Nu}_l = \omega_{1w} \cos\varphi \cdot \sin\psi + \omega_{2w} \cos\psi. \tag{19}$$

From relationships (18), (19) we obtain

$$\mathrm{Nu}/\mathrm{Nu}_l = 1 + \mathrm{Nu}_l\{1{,}32\cdot 10^{-4}(\mathrm{Gr}/\mathrm{Re})[1 - \exp(-40X)]\cos\psi\} - 9.1\cdot 10^{-6}\,\mathrm{Ra}[1 - \exp(-100X)]^3 \cos\varphi \cdot \sin\psi. \tag{20}$$

Figure 3 shows the change in the relative value of the Nusselt number at the upper ($\varphi = 0$, solid curves) and lower ($\varphi = \pi$, dashed curves) generatrices of the heated tube as a function of the angle ψ between the vertical and the axis of the tube. Within the limits of the value of the angle $0 \leq \psi < \pi/2$, there is flow from the bottom upwards and with $\pi/2 < \psi \leq \pi$, flow from the top downward. Curves 1-5 relate to the case of the greatest effect of thermogravitation along the length, i.e., to a value of $X > 0.07$. With a decrease in X the effect of thermogravitation decreases, as follows from all the dependences given. Curves 1 were obtained with the values $(Gr/Re) = 250$, $Ra = 5\cdot 10^3$. With all values of the angle ψ, with the exception of the immediate vicinity of the vertical position within limits not exceeding 10°, there is a considerable difference in the heat transfer at the upper and lower generatrices. Under these circumstances, the heat transfer at the lower generatrix takes on a maximal value (or a maximal value at the upper generatrix) somewhere in a middle position of the tube between the vertical and the horizontal. With a decrease in the Prandtl number (actually, a decrease in the Ra number), the difference in the heat transfer at the upper and lower generatrices decreases. Curves 1-3 relate to exactly the same value of Gr/Re, but to different values of Ra. The value of Ra for curves 2 is 4 times lower than for the curves 1, while, for curves 3, the Ra number $Ra < 10^2$. Curves 3 actually correspond to the viscosity-gravitation flow of liquid metals. For the given set of parameters, the local heat transfer of liquid metals in horizontal tubes is insensitive to the effect of thermogravitation, while, in vertical tubes, this effect will be considerable. It must be recalled that, here, there is a strong deformation of the profiles of the velocity in horizontal and inclined tubes, as is shown in Figs. 1 and 2. Curves 4 and 5 show the change in the heat transfer with the same values of Gr and Pr as for curves 1, but with different values of Re. A change in the Re number has an effect on the heat transfer in vertical tubes. Curves 4 were plotted for a value of $(Gr/Re) = 500$, and curves 5 for $(Gr/Re) = 125$. The effect of the reduced length is illustrated by curves 6, which were plotted for $X = 5.2\cdot 10^{-3}$, and other parameters corresponding to curves 4. It can be seen that the heat transfer in inclined tubes varies in a very specific manner. Specifically, with some inclinations, there is the possibility of situations where the heat transfer along one generatrix practically does not change, while along the other generatrix it varies considerably.

Figure 4 shows a comparison of calculations of the local heat transfer in accordance with the dependence (20) with experimental data, obtained with the viscosity-gravitational flow of water in horizontal [8] and vertical [9] tubes. Line 1 corresponds to the case of laminar flow. The experimental data 1 on heat transfer at the upper and the data 2 for heat transfer at the lower generatrices of a horizontal tube were obtained with $Ra = 7\cdot 10^6$. The corresponding curves 4, 5 were calculated for the same value of Ra. The experimental data 3, obtained in a vertical tube, and the calculated curve 6, relate to the value $(Gr/Re) = 1.2\cdot 10^3$. On the basis of comparison with the experimental data, a conclusion can be drawn

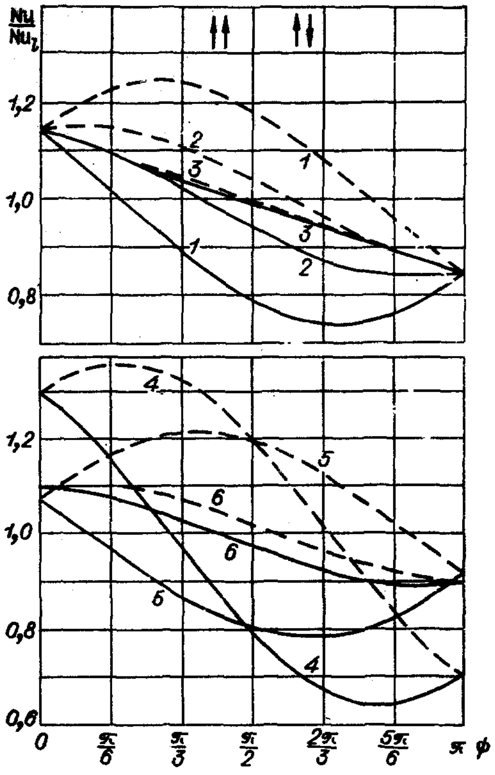

Fig. 3

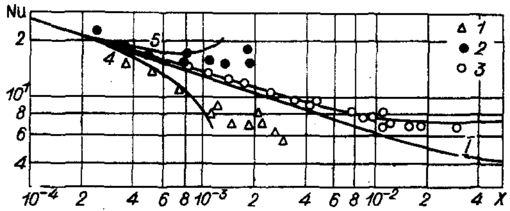

Fig. 4

with respect to the correctness of the calculation, within the validity of the assumption of smallness.

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